UNCERTAINTY OF PEAK NOISE LEVEL MEASUREMENTS – USING STATISTICAL TOLERANCE LIMIT

Krzysztof Rudno-Rudzinski

Wroclaw University of Technology
Institute of Telecommunication, Teleinformatics and Acoustics
Wyspianskiego 27, 50370 Wroclaw, Poland
krzysztof.rudno-rudzinski@pwr.wroc.pl

ABSTRACT

According to the Noise at Work Directive 2003/10/EC, an assessment of noise measurement results should take into account measurement inaccuracies.

Measurement inaccuracies are expressed by the measurement uncertainty. The possibility that values larger or smaller than the value recognized as the measurement result may occur is accepted.

The peak sound pressure level is defined as the maximum value of the C-weighted instantaneous noise pressure level. By definition, there is no higher value than the maximum one. Thus, the peak level measurement uncertainty cannot be determined in the usual way, i.e. through a confidence interval, because the maximum peak pressure level is a random variable, not a random variable distribution parameter.

The paper presents arguments for the probabilistic approach to the maximum peak level value determination. It is proposed to use a statistical tolerance interval for the measurement uncertainty determination by the sampling method.

The proposed approach is a compromise: the maximum peak noise level value is sought while at the same time the possibility that values higher than the measured one may occur is accepted.

1. INTRODUCTION

The use of *uncertainty* as a measure of measurement quality has become established in recent years. The key international organizations have approved the unified principles of calculating and expressing the measurement uncertainty, contained in the publication 'Guide to the Expression of Uncertainty in Measurement' [1]. This has been reflected in the requirement that test laboratories should have and apply measurement uncertainty estimation procedures [2]. The measurement uncertainty determination according to the Guide has become widespread in acoustic regulations and standards. According to the Noise at Work Directive 2003/10/EC concerning the minimum health and safety requirements regarding the exposure of workers to the risks arising from physical agents (noise), '…the assessment of the measurement results shall take into account the measurement inaccuracies determined in accordance with metrological practice' [3].

Relevant regulations and standards specify the methods of testing and measuring environmental factors harmful to human health. The basic predictors of noise risk, defined in Directive 2003/10/EC are: the peak sound pressure (p_{peak}), i.e. maximum value of the 'C'-frequency weighted instantaneous noise pressure; and the noise exposure level for a working day or a week, defined in the international standard ISO 1999 [4]. In Poland L_{Cpeak} (the maximum value of the 'C'-frequency weighted instantaneous noise pressure level) is used instead of p_{peak}. The maximum 'A'-weighted sound level A (L_{Amax}) is also applied in occupational noise assessment.

In the information annex to the standard ISO 9612, containing guidelines for measuring and assessing noise exposure in the work environment [5], one can find a method of determining the uncertainty of measurement of the 'A'-weighted equivalent acoustic pressure level, being the basis for the noise exposure and the noise exposure level calculations. But neither in this standard nor in any other work occupational noise standards one can find a method of determining the measurement uncertainty for L_{Cpeak} (and L_{Amax}). This paper presents an attempt to estimate the measurement uncertainty for L_{Cpeak}.

2. UNCERTAINTY OF SOUND PRESSURE LEVEL MEASUREMENTS

Measurement uncertainty is a parameter associated with the measurement result, characterizing the dispersion of values [1]. Generally, two types of uncertainty are distinguished. They differ in the way the uncertainty is calculated: an A-type method – by the statistical analysis of a series of individual observations and a B-type method – by other methods than the statistical analysis of a series of observations. In method A the error statistics are not known *a priori* and so the measure of the dispersion of values, i.e. standard deviation σ_A, is proportional to $1/n$ (n – the number of measurements). If the error in consecutive measurements remains constant (this applies to, e.g. errors introduced by measuring instruments), it is calculated by the method B. The uncertainty of the type A is associated with the sampling method since when the number of measurements (samples) is increased, the scattering of results decreases.

The measurement uncertainty is determined through the measure of the scattering of results, i.e. standard deviation. In practice extended uncertainty U is used, which specifies the interval around the measurement result, covering a large part of p of the probability distribution described by this result, where p is the level of confidence or probability of covering the interval [1].

Let us consider a situation in which using an integrating sound level meter one performs n equivalent sound level measurements $L_1, L_2, \ldots L_i, \ldots L_n$ with averaging time T_A. The total measurement time is

$$T = nT_A < T_e \tag{1}$$

where T_e is the total noise exposure time.

The measurement uncertainty determination is considered as the estimation of a random variable probability distribution parameter. The parameter is usually an average. When measuring the equivalent level by the sampling method one assumes [5]

$$L_{Aeq,T} = \overline{L} + 0.115 \cdot s^2 \tag{2}$$

as the measurement result, where

$$\overline{L} = \frac{\sum_n L_i}{n} \tag{3}$$

is the average, and

$$s^2 = \frac{\sum_n (L_i - \overline{L})^2}{n-1} \tag{4}$$

– the standard deviation of measurement results.

Component $0.115 \cdot s^2$ stems from the fact that when determining $L_{Aeq,T}$ one averages not the acoustic pressure values but the squares of the acoustic pressure. For the normal distribution of acoustic pressure levels the squares of acoustic pressures have a log-normal distribution.

The confidence interval for $L_{Aeq,T}$ at confidence level $p = 1 - \alpha$ is defined by the limits

$$\left[L_{Aeq,T} - \sqrt{\frac{s^2}{n} + \frac{0.026 \cdot s^4}{n-1}} \cdot t_{n-1,\alpha}, L_{Aeq,T} + \sqrt{\frac{s^2}{n} + \frac{0.026 \cdot s^4}{n-1}} \cdot t_{n-1,\alpha} \right] \tag{5}$$

where $t_{n-1,\alpha}$ is the value of the Student's t-distribution variable for n-1 degrees of freedom and a selected α [5].

3. PEAK LEVEL MEASUREMENTS

The situation is totally different when one measures peak level L_{Cpeak} by the sampling method. As the measurement result one assumes the highest of the i measured values of L_{Cpeak}, i.e.

$$L_{Cpeak,\max} = \max\{L_{Cpeak,i}\} \tag{6}$$

without referring to its statistical variation. No random variable distribution parameter estimation occurs here as in the case of measuring $L_{Aeq,T}$. As the result, it is not possible to apply a typical procedure of calculating the confidence level for the estimated parameter of population.

When applying the sampling method one should expect that when measurements are repeated, levels higher than the previously measured $L_{Cpeak,\max}$ may occur. Then one should adjust the level to $L_{Cpeak,\max 1} > L_{Cpeak,\max}$. Consequently, only after measurements in the whole noise exposure time have been performed one can be sure of the maximum value.

The postulate that measurements should be performed in the whole noise exposure time is in contradiction with the sampling method. The sampling method is commonly used.. Technically it is possible to eliminate it by equipping all employees with personal noise dosimeters, but at the moment this solution seems to be unrealistic. The use of dosimeters is not a solution to the main problem which is the protection against adverse effects of noise in the future.

The main purpose of measuring the occupational noise is not to find out what noise exposures occurred in the past, but to predict a future noise exposure. The dosimetry allows one to accurately determine the history of the noise impact on the employee whereby it can be the basis for claims for compensation for deterioration of hearing as a result of exposure to noise in the work environment, but it does not solve the noise hazard prediction problem. Thus the gist of the problem is the prediction of noise indices, including their maximum values.

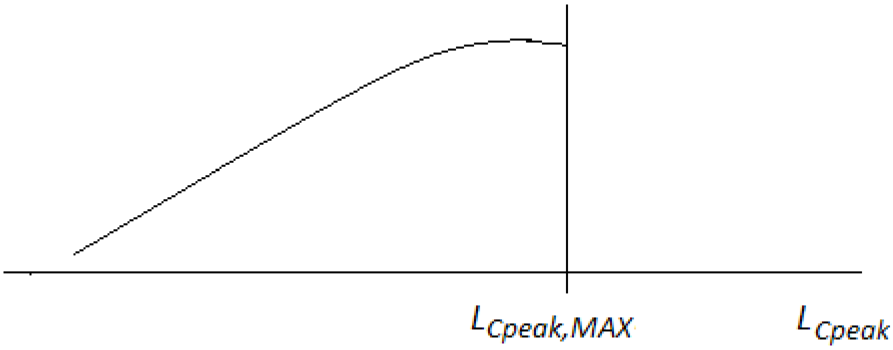
4. STATISTICAL TOLERANCE LIMITS OF PEAK LEVEL

Regulations require that the maximum value of instantaneous acoustic pressure level is determined. From the statistical point of view, this requirement is based on the assumption that the maximum level probability density function is equal to zero above the sought maximum value:

$$p(L_{C,peak} > L_{Cpeak,MAX}) = 0 \tag{7}$$

It would be very difficult to substantiate this assumption for many of the sources which occur in the work environment. A continuous probability density function, perhaps with a sharp slope above a certain value, would better model the reality (Fig.1). Then the maximum value measurement should be replaced with the measurement of a quasi-maximum value being a specified quantile of a random variable distribution (Fig.2).

Staying with maximum value determination, the measured value of $L_{Cpeak,\max}$ and an interval in which maximum value measurement results can appear for successive measurements should be adopted. For $L_{Cpeak,\max}$ determined by the sampling method we are interested in determining $L_{Cpeak,q} > L_{Cpeak,\max}$, which is not exceeded by given part q of the population of all the measurement results. This part should be great enough for the occurrence probability of the value higher than $L_{Cpeak,M}$ to be close to zero.



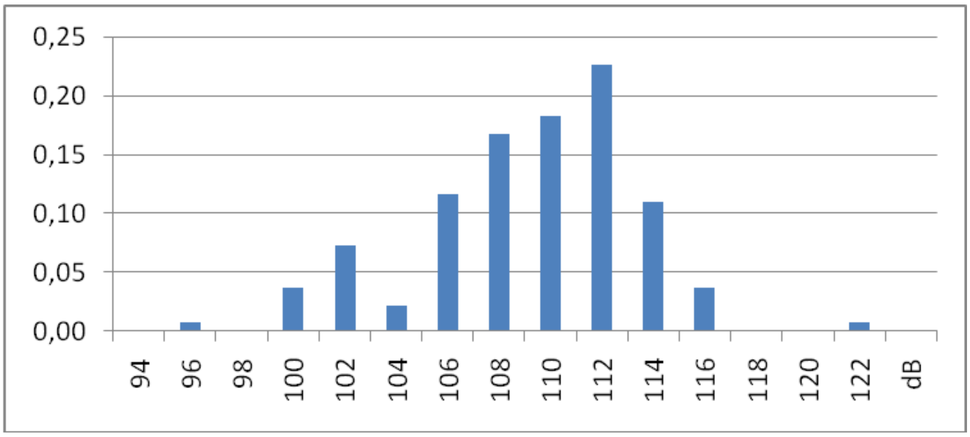


Figure 1. *Probability density function of* L_{Cpeak} *according to eq. (7)(up)and the histogram of measured* L_{Cpeak} *values for orchestra music (down)*

The above formulation of the problem leads to statistical limits of tolerance. The latter are the limits in which one expects to find part *q* of the population. The quasi-maximum value is a value which is not exceeded in *q* [%] measurements with confidence 1-α.

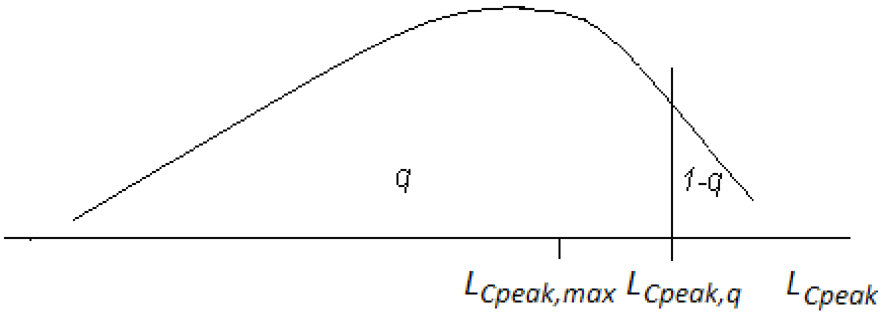


Figure 2. *Quasi-maximum value* $L_{Cpeak,q}$ *as q-quantile of* L_{Cpeak} *distribution*.

5. DETERMINATION OF STATISTICAL TOLERANCE LIMIT

In the considered case the aim is to determine the upper tolerance limit for $L_{Cpeak,\max}$. The estimator $L_{Cpeak,q,1-\alpha}$ for an unknown value of $L_{Cpeak,q}$, covering part q of the population, will be determined at the confidence level $1-\alpha$:

$$\Pr(L_{Cpeak,q,1-\alpha} \geq L_{Cpeak,q}) = 1-\alpha \tag{8}$$

The estimator of the one-sided upper limit of tolerance is given by the equation [6]

$$L_{Cpeak,q,1-\alpha} = \overline{L}_{Cpeak} + k_1 \cdot s \tag{9}$$

where $\overline{L}_{Cpeak} = \frac{\sum_{i=1}^{n} L_{Cpeak,i}}{n}$ is the average, $s = \frac{\sum_{i=1}^{n} (L_{Cpeak,i} - \overline{L}_{Cpeak})^2}{(n-1)}$ – standard deviation and [7]

$$k_1 = \frac{t(n-1,\delta;1-\alpha)}{\sqrt{n}} \tag{10}$$

while $t(n-1,\delta;1-\alpha)$ is a value of variable t for Student's noncentral t-distribution with $n-1$ degrees of freedom for $\Pr(t \leq t_{1-\alpha}) = 1-\alpha$ and noncentrality parameter δ is expressed by

$$\delta = K_q \cdot \sqrt{n} \tag{11}$$

The coefficient K_q is a quantile of the order q of the normalized Gaussian distribution

$$N(0,1,q) = \int_{-\infty}^{K_q} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{x^2}{2}} dx = q \tag{12}$$

In order to calculate the coefficient k_1 one can use an approximate formula [6] or an NDC calculator [8].

Part q of measurement results which should not be higher than quasi-maximum value L_{Cpeak} can be adopted on the basis of the L_{Cpeak} measurement results.

For this purpose one can use the set $\{L_{Cpeak,i}\}$ of all the maximum level measurement results. Assuming a normal distribution of L_{Cpeak} with the average $\overline{L}_{Cpeak}$ and the standard deviation s, q can be calculated as follows (Pr = probability)

$$\Pr(L_{Cpeak} \leq L_{Cpeak,\max}) = \int_{-\infty}^{L_{Cpeak,\max}} \frac{1}{s\sqrt{2\pi}} \cdot e^{-\frac{(L_{Cpeak} - \overline{L}_{Cpeak})^2}{2s^2}} dL_{Cpeak} = q \tag{13}$$

where $\overline{L}_{Cpeak} = \frac{\sum_{i=1}^{n} L_{Cpeak,i}}{n}$ and $s = \frac{\sum_{i=1}^{n} (L_{Cpeak,i} - \overline{L}_{Cpeak})^2}{(n-1)}$.

The calculated q will be the basis for the tolerance limit calculations for the whole population of the measurement results.

6. PEAK LEVEL MEASUREMENT UNCERTAINTY

The calculated $L_{Cpeak,q,1-\alpha}$ fixes the limit of the tolerance interval containing 100q per cent of all the L_{Cpeak} measurement results at the confidence level 1-α.

The difference between the tolerance limit $L_{Cpeak,q,1-\alpha}$ and the measured maximum value $L_{Cpeak,\max}$ can be regarded as a measure of the measurement uncertainty:

$$U = L_{Cpeak,q,1-\alpha} - L_{Cpeak,\max} \tag{14}$$

As the number of measurements is increased, $L_{Cpeak,\max}$ approaches infinity (assuming normal distribution of L_{Cpeak}). For the abovementioned way of calculating $L_{Cpeak,q,1-\alpha}$ U will always be positive. One can expect that it will decrease as n increases, which is typical for the A-type uncertainty.

As the result of the peak level measurement one should give the measured value of $L_{Cpeak,\max}$ and interval U calculated from equation (14), including the information that the interval covers $100 \cdot q$ per cent of the results with the probability 1-α. The probability should be selected similarly as the confidence level, i.e. 0.90 or 0.95. The information about the part of the population contained in the interval is an additional piece of information which allows one to assess the probability of exceeding the measured value.

7. EXEMPLARY MEASUREMENT DATA

During a concert of the symphonic orchestra, lasting for 2 hours, L_{Cpeak} values were recorded at every 15 seconds in seven locations among the musicians (Fig. 3).

Figure 3. *Microphone layout*.

Table I shows the results of calculations of the average $\overline{L}_{Cpeak}$ and the standard deviation s and the read out value of $L_{Cpeak,\max}$ for the particular microphones. The value of q was calculated from formula (13) and then $L_{Cpeak,q,1-\alpha}$ and measurement uncertainty U, at a confidence level of 0.95, were calculated.

Table 1. Measurement uncertainty for L_{Cpeak} in symphonic orchestra

	M1: Cello (near brasses)	M2:Trombone (in front of timpani)	M3: Timpani	M4:Violin	M5:Flute (near timpani)	M6:Trumpet (in front of bass tuba)	M7:Viola
$L_{Cpeak,\max}$	127.7	133.6	134.1	122.7	126.1	125.5	120.8
$\overline{L}_{Cpeak}$	100.3	98.6	97.3	99.5	101.0	99.3	99.5
s	10.6	11.3	11.5	9.6	10.5	11.0	9.5
q	0.995	0.999	0.999	0.992	0.992	0.991	0.987
$L_{Cpeak,q,1-\alpha}$	129.4	135.7	136.3	124.2	127.7	127.2	122.2
U	1.7	2.1	2.2	1.5	1.6	1.7	1.4

8. CONCLUSIONS

In order to be able to predict the noise exposure in the work environment on the basis of measurement results one must determine the measurement uncertainty not only for the equivalent sound pressure level, but also for the peak ‘C’-weighted sound pressure level L_{Cpeak}.

The typical way of expressing measurement uncertainty through confidence interval cannot be applied to the L_{Cpeak}. The presented method of determining the measurement uncertainty consists of combining the highest measured peak level $L_{Cpeak,\max}$ and the upper tolerance limit for L_{Cpeak}.

The method was applied to the results of measurements in the work environment of opera musicians. The biggest uncertainties – over 2 dB – have been found out for timpani.

9. REFERENCES

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